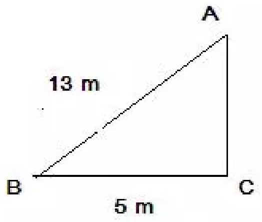
Chapter 13. Pythagoras Theorem [Proof and Simple Applications with Converse]

Exercise 13(A)

Solution 1:

The pictorial representation of the given problem is given below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

(i)Here, AB is the hypotenuse. Therefore applying the Pythagoras theorem we get,

$AB^2 = BC^2 + CA^2$

$13^2 = 5^2 + CA^2$

$CA^2 = 13^2 - 5^2$

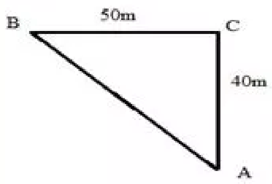
$CA^2 = 144$

$CA = 12$ m

Therefore, the distance of the other end of the ladder from the ground is 12m

Solution 2:

Here , we need to measure the distance AB as shown in the figure below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore , in this case

$AB^2 = BC^2 + CA^2$

$AB^2 = 50^2 + 40^2$

$AB^2 = 2500 + 1600$

$AB^2 = 4100$

$AB = 64.03$

Therefore the required distance is 64.03 m.

Solution 3:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔPQS and applying Pythagoras theorem we get,

$PQ^2 = PS^2 + QS^2$

$10^2 = PS^2 + 6^2$

$PS^2 = 100 - 36$

$PS = 8$

Now, we consider the ΔPRS and applying Pythagoras theorem we get,

$PR^2 = RS^2 + PS^2$

$PR^2 = 15^2 + 8^2$

$PR = 17$

The length of PR 17 cm

Solution 4:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔBDC and applying Pythagoras theorem we get,

$$DB^2 = DC^2 + BC^2$$
$$DB^2 = 12^2 + 3^2$$
$$DB^2 = 144 + 9$$
$$DB^2 = 153$$

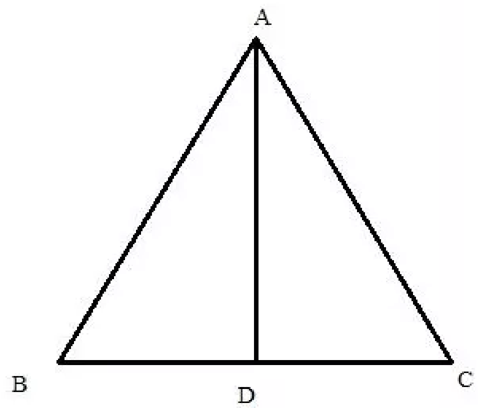
Now, we consider the ΔABD and applying Pythagoras theorem we get,

$$DA^2 = DB^2 + BA^2$$
$$13^2 = 153 + BA^2$$
$$BA^2 = 169 - 153$$
$$BA = 4$$

The length of AB is 4 cm.

Solution 5:

Since ABC is an equilateral triangle therefore, all the sides of the triangle are of same measure and the perpendicular AD will divide BC in two equal parts.



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, we consider the ΔABD and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = 100^2 - 5^2 \quad \left[\begin{array}{l}\text{Given}, BC = 10 \text{ cm} = AB, \\ BD = \frac{1}{2}BC\end{array}\right]$$

Therefore, the length of AD is 8.7 cm

$$AD^2 = 100 - 25$$
$$AD^2 = 75$$
$$AD = 8.7$$

Solution 6:

We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABO, and applying Pythagoras theorem we get,

$$AB^2 = AO^2 + OB^2$$
$$AO^2 = AB^2 - OB^2$$
$$AO^2 = AB^2 - (BC + OC)^2$$
$$[\text{Let}, OC = x]$$
$$AO^2 = AB^2 - (BC + x)^2 \qquad ......(i)$$

First, we consider the ΔACO, and applying Pythagoras theorem we get,

$$AC^2 = AO^2 + x^2$$
$$AO^2 = AC^2 - x^2 \qquad ......(ii)$$

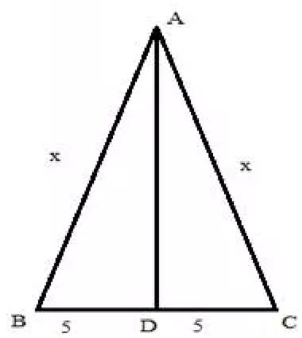
Now, from (i) and(ii),

$$AB^2 - (BC + x)^2 = AC^2 - x^2$$
$$8^2 - (6 + x)^2 = 3^2 - x^2 \qquad [\text{Given}, AB = 8cm, BC = 8cm \text{ and } AC = 3cm]$$
$$x = 1\frac{7}{12}cm$$

Therefore, the length of OC will be $1\frac{7}{12}$ cm

Solution 7:

Here, the diagram will be,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABC is an isosceles triangle, therefore perpendicular from vertex will cut the base in two equal segments.

First, we consider the ΔABD, and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$
$$AD^2 = x^2 - 5^2$$
$$AD^2 = x^2 - 25$$
$$AD = \sqrt{x^2 - 25} \qquad ......(i)$$

Now,

$$\text{Area} = 60$$
$$\frac{1}{2} \times 10 \times AD = 60$$
$$\frac{1}{2} \times 10 \times \sqrt{x^2 - 25} = 60$$
$$x = 13$$

Therefore, x is 13cm

Solution 8:

Let, the sides of the triangle be, x, $\sqrt{2}x$ *and* x

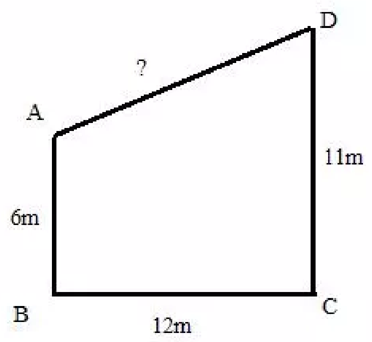
Now, $x^2 + x^2 = 2x^2 = \left(\sqrt{2}x\right)^2$ ......(i)

Here, in (i) it is shown that, square of one side of the given triangle is equal to the addition of square of other two sides. This is nothing but Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore, the given triangle is a right angled triangle.

Solution 9:

The diagram of the given problem is given below,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, $11 - 6 = 5m$ (Since DC is perpendicular to BC)

base = 12m

Applying Pythagoras theorem we get,

$$\text{hypotenuse}^2 = 5^2 + 12^2$$

$$h^2 = 25 + 144$$

$$h^2 = 169$$

$$h = 13$$

Therefore, the distance between the tips will be 13m

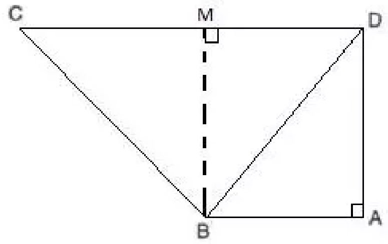
Solution 10:

Take M be the point on CD such that AB = DM.

So DM = 7cm and MC = 10 cm

Join points B and M to form the line segment BM.

So BM || AD also BM = AD.



In right-angled ΔBAD

$BD^2 = AD^2 + BA^2$

$(25)^2 = AD^2 + (7)^2$

$AD^2 = (25)^2 - (7)^2$

$AD^2 = 576$

$AD = 24$

In right-angled ΔCMB

$CB^2 = CM^2 + MB^2$

$CB^2 = (10)^2 + (24)^2$ $\quad [MB = AD]$

$CB^2 = 100 + 576$

$CB^2 = 676$

$CB = 26 cm$

Solution 11:

Given that AX:XB = 1:2.

Let n be the common multiple for which this proportion gets satisfied.

So, AX = 1(n) and XB = 2(n)

$AX + XB = 1(n) + 2(n)$

$\Rightarrow AB = n + 2n$

$\Rightarrow 12 = 3n$

$\Rightarrow n = 4$

AX = 1(n) = 4 and XB = 2(n) = 8

In ΔABC,

$XY \parallel BC$

$\frac{AB}{AX} = \frac{AC}{AY} = \frac{BC}{XY}$

$\Rightarrow \frac{AB}{AX} = \frac{AC}{AY}$

$\Rightarrow \frac{12}{4} = \frac{AC}{8}$

$\Rightarrow AC = 24cm$

In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (24)^2 = (12)^2 + BC^2$

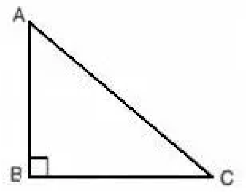
$\Rightarrow BC^2 = (24)^2 - (12)^2$

$\Rightarrow BC^2 = 576 - 144$

$\Rightarrow BC^2 = 432$

$\Rightarrow BC = 12\sqrt{3} cm$

Solution 12:



(i) In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (x+6)^2 = (x-3)^2 + (x+4)^2$

$\Rightarrow (x^2 + 12x + 36) = (x^2 - 6x + 9) + (x^2 + 8x + 16)$

$\Rightarrow x^2 - 10x - 11 = 0$

$\Rightarrow (x-11)(x+1) = 0$

$\Rightarrow x = 11$ or $x = -1$

But length of the side of a triangle can not be negative.

$\Rightarrow x = 11cm$

$\therefore AB = (x-3) = (11-3) = 8cm$

$BC = (x+4) = (11+4) = 15cm$

$AC = (x+6) = (11+6) = 17cm$

(ii) In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (4x+5)^2 = (x)^2 + (4x+4)^2$

$\Rightarrow (16x^2 + 40x + 25) = (x^2) + (16x^2 + 32x + 16)$

$\Rightarrow x^2 - 8x - 9 = 0$

$\Rightarrow (x-9)(x+1) = 0$

$\Rightarrow x = 9$ or $x = -1$

But length of the side of a triangle can not be negative.

$\Rightarrow x = 9cm$

$\therefore AB = x = 9cm$

$BC = (4x+4) = (36+4) = 40cm$

$AC = (4x+5) = (36+5) = 41cm$

Exercise 13(B)

Solution 1:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABD and applying Pythagoras theorem we get,

$AB^2 = AD^2 + BD^2$

$c^2 = h^2 + (a-x)^2$

$h^2 = c^2 - (a-x)^2$ ......(i)

First, we consider the ΔACD and applying Pythagoras theorem we get,

$AC^2 = AD^2 + CD^2$

$b^2 = h^2 + x^2$

$h^2 = b^2 - x^2$ ......(ii)

From (i) and (ii) we get,

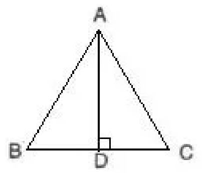
$c^2 - (a-x)^2 = b^2 - x^2$

$c^2 - a^2 - x^2 + 2ax = b^2 - x^2$

$c^2 = a^2 + b^2 - 2ax$

Hence Proved

Solution 2:



In equilateral Δ ABC, AD ⊥ BC.

Therefore, BD = DC = x/2 cm.

In right-angled ΔADC

$AC^2 = AD^2 + DC^2$

$\Rightarrow (x)^2 = AD^2 + \left(\frac{x}{2}\right)^2$

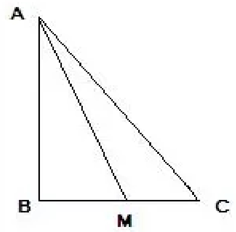
$\Rightarrow AD^2 = (x^2) - \left(\frac{x}{2}\right)^2$

$\Rightarrow AD^2 = \left(\frac{x}{2}\right)^2$

$\Rightarrow AD = \left(\frac{x}{2}\right) cm$

Solution 3:

The pictorial form of the given problem is as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABM and applying Pythagoras theorem we get,

$AM^2 = AB^2 + BM^2$

$AB^2 = AM^2 - BM^2 \quad ......(i)$

Now, we consider the ΔABC and applying Pythagoras theorem we get,

$AC^2 = AB^2 + BC^2$

$AB^2 = AC^2 - BC^2 \quad ......(ii)$

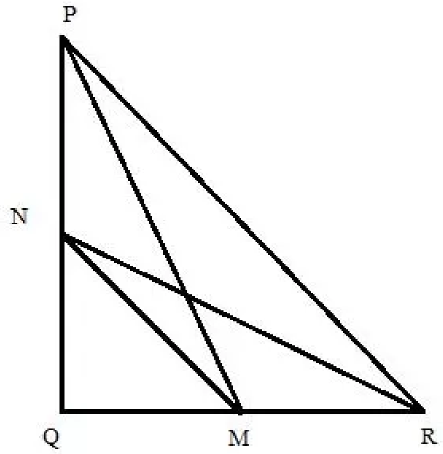
From (i) and (ii) we get,

$AM^2 - BM^2 = AC^2 - BC^2$

$AM^2 + BC^2 = AC^2 + BM^2$

Hence Proved

Solution 4:



We draw , PM,MN,NR

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
Since, M andN are the mid-points of the sides QR and PQ respectively, therefore, PN=NQ,QM=RM
(i)
First, we consider the ΔPQM, and applying Pythagoras theorem we get,

$$PM^2 = PQ^2 + MQ^2$$
$$= (PN + NQ)^2 + MQ^2$$
$$= PN^2 + NQ^2 + 2PN.NQ + MQ^2$$
$$= MN^2 + PN^2 + 2PN.NQ \qquad \begin{bmatrix}\text{From, } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2\end{bmatrix} \ldots\ldots(i)$$

Now, we consider the ΔRNQ, and applying Pythagoras theorem we get,

$$RN^2 = NQ^2 + RQ^2$$
$$= NQ^2 + (QM + RM)^2$$
$$= NQ^2 + QM^2 + RM^2 + 2QM.RM$$
$$= MN^2 + RM^2 + 2QM.RM \qquad \ldots\ldots(ii)$$

Adding (*i*) and (*ii*) we get,

$$PM^2 + RN^2 = MN^2 + PN^2 + 2PN.NQ + MN^2 + RM^2 + 2QM.RM$$
$$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN.NQ + 2QM.RM$$
$$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2)$$
$$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$$
$$PM^2 + RN^2 = 5MN^2$$

Hence Proved

(ii)
We consider the ΔPQM, and applying Pythagoras theorem we get,

$$PM^2 = PQ^2 + MQ^2$$
$$4PM^2 = 4PQ^2 + 4MQ^2 \qquad \begin{bmatrix}\text{Multiplying both} \\ \text{sides by 4}\end{bmatrix}$$
$$4PM^2 = 4PQ^2 + 4\cdot\left(\frac{1}{2}QR\right)^2 \qquad \left[MQ = \frac{1}{2}QR\right]$$
$$4PM^2 = 4PQ^2 + 4\cdot\frac{1}{4}QR^2$$
$$4PM^2 = 4PQ^2 + QR^2$$

Hence Proved

(iii)

We consider the ΔRQN, and applying Pythagoras theorem we get,

$$RN^2 = NQ^2 + RQ^2$$

$$4RN^2 = 4NQ^2 + 4QR^2 \qquad \begin{bmatrix}\text{Multiplying both} \\ \text{sides by 4}\end{bmatrix}$$

$$4RN^2 = 4QR^2 + 4\cdot\left(\frac{1}{2}PQ\right)^2 \qquad \left[NQ = \frac{1}{2}PQ\right]$$

$$4RN^2 = 4QR^2 + 4\cdot\frac{1}{4}PQ^2$$

$$4RN^2 = PQ^2 + 4QR^2$$

Hence Proved

(iv)

First, we consider the ΔPQM, and applying Pythagoras theorem we get,

$$PM^2 = PQ^2 + MQ^2$$
$$= (PN + NQ)^2 + MQ^2$$
$$= PN^2 + NQ^2 + 2PN.NQ + MQ^2$$
$$= MN^2 + PN^2 + 2PN.NQ \qquad \begin{bmatrix}\text{From, } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2\end{bmatrix} \ldots\ldots (i)$$

Now, we consider the ΔRNQ, and applying Pythagoras theorem we get,

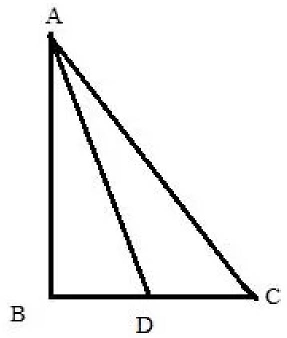
$$RN^2 = NQ^2 + RQ^2$$
$$= NQ^2 + (QM + RM)^2$$
$$= NQ^2 + QM^2 + RM^2 + 2QM.RM$$
$$= MN^2 + RM^2 + 2QM.RM$$
$$\ldots\ldots (ii)$$

Adding (*i*) and (*ii*) we get,

$$PM^2 + RN^2 = MN^2 + PN^2 + 2PN.NQ + MN^2 + RM^2 + 2QM.RM$$
$$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN.NQ + 2QM.RM$$
$$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2)$$
$$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$$
$$PM^2 + RN^2 = 5MN^2$$
$$4\left(PM^2 + RN^2\right) = 4\cdot 5\cdot (NQ^2 + MQ^2)$$
$$4\left(PM^2 + RN^2\right) = 4\cdot 5\cdot\left[\left(\frac{1}{2}PQ\right)^2 + \left(\frac{1}{2}QR\right)^2\right]$$
$$\left[\because NQ = \frac{1}{2}PQ, MQ = \frac{1}{2}QR\right]$$
$$4\left(PM^2 + RN^2\right) = 5PR^2$$

Hence Proved

Solution 5:



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

In triangle ABC, $\angle B = 90^\circ$ and D is the mid-point of BC. Join AD. Therefore, BD=DC

First, we consider the ΔADB, and applying Pythagoras theorem we get,

$$AD^2 = AB^2 + BD^2$$

$$AB^2 = AD^2 - BD^2 \quad ......(i)$$

Similarly, we get from rt. angle triangles ABC we get,

$$AC^2 = AB^2 + BC^2$$

$$AB^2 = AC^2 - BC^2 \quad ......(ii)$$

From (i) and (ii) ,

$$AC^2 - BC^2 = AD^2 - BD^2$$

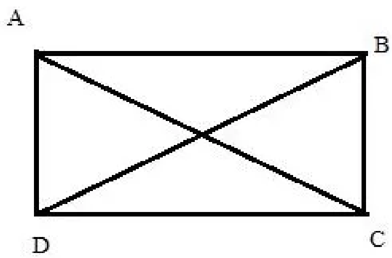
$$AC^2 = AD^2 - BD^2 + BC^2$$

$$AC^2 = AD^2 - CD^2 + 4CD^2 \quad \left[BD = CD = \frac{1}{2}BC\right]$$

$$AC^2 = AD^2 + 3CD^2$$

Hence proved.

Solution 6:



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABCD is a rectangle angles A,B,C and D are rt. angles.

First, we consider the ΔACD, and applying Pythagoras theorem we get,

$$AC^2 = DA^2 + CD^2 \quad ......(i)$$

Similarly, we get from rt. angle triangle BDC we get,

$$BD^2 = BC^2 + CD^2$$

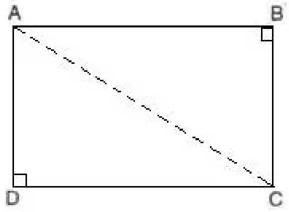
$$= BC^2 + AB^2 \quad \left[\begin{array}{l}\text{In a rectangle , opposite} \\ \text{sides are equal,} \therefore CD = AB\end{array}\right] \quad ......(ii)$$

Adding (i) and (ii) ,

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$$

Hence proved.

Solution 7:



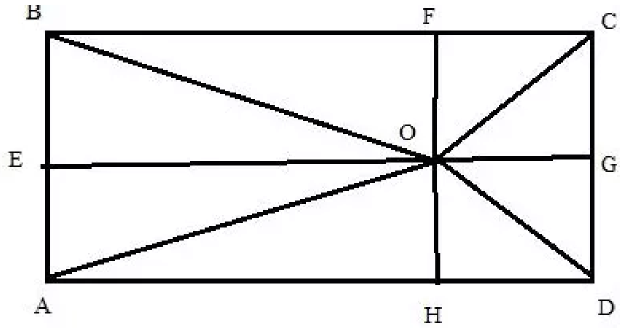
In quadrilateral ABCD, $\angle B = 90^\circ$ and $\angle D = 90^\circ$.
So, ΔABC and ΔADC are right-angled triangles.

In ΔABC using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$\Rightarrow AB^2 = AC^2 - BC^2 ....(i)$

In ΔADC, using Pythagoras theorem,
$AC^2 = AD^2 + DC^2 ....(ii)$

$LHS = 2AC^2 - AB^2$
$= 2AC^2 - (AC^2 - BC^2)$ [from(i)]
$= 2AC^2 - AC^2 + BC^2$
$= AC^2 + BC^2$
$= AD^2 + DC^2 + BC^2$ [from(ii)]
$= RHS$

Solution 8:



Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD. Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Using Pythagorean theorem we have from the above diagram:

$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$

$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$

$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$

$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$

Adding these equalities we get:

$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$

$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$

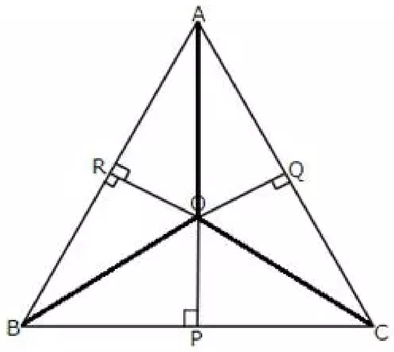
From which we prove that for any point within the rectangle there is the relation

$OA^2 + OC^2 = OB^2 + OD^2$

Hence Proved.

Solution 9:

Here, we first need to join OA, OB, and OC after which the figure becomes as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides. First, we consider the ΔARO and applying Pythagoras theorem we get,

$$AO^2 = AR^2 + OR^2$$

$$AR^2 = AO^2 - OR^2 \quad \ldots\ldots (i)$$

Similarly, from triangles, BPO,COQ,AOQ,CPO and BRO we get the following results,

$$BP^2 = BO^2 - OP^2 \quad \ldots\ldots (ii)$$

$$CQ^2 = OC^2 - OQ^2 \quad \ldots\ldots (iii)$$

$$AQ^2 = AO^2 - OQ^2 \quad \ldots\ldots (iv)$$

$$CP^2 = OC^2 - OP^2 \quad \ldots\ldots (v)$$

$$BR^2 = OB^2 - OR^2 \quad \ldots\ldots (vi)$$

Adding (*i*), (*ii*) and (*iii*),we get

$$AR^2 + BP^2 + CQ^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2 \quad \ldots\ldots (vii)$$

Adding (*iv*), (*v*) and (*vi*),we get ,

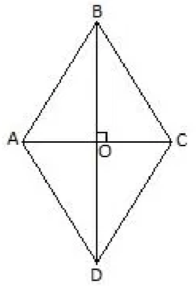
$$AQ^2 + CP^2 + BR^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2 \quad \ldots\ldots (viii)$$

From (*vii*) and (*viii*), we get,

$AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$

Hence proved.

Solution 10:



Diagonals of the rhombus are perpendicular to each other.

In quadrilateral ABCD, $\angle AOD = \angle COD = 90^\circ$.
So, ΔAOD and ΔCOD are right-angled triangles.

In ΔAOD using Pythagoras theorem,

$AD^2 = OA^2 + OD^2$

$\Rightarrow OA^2 = AD^2 - OD^2 ....(i)$

In ΔCOD using Pythagoras theorem,

$CD^2 = OC^2 + OD^2$

$\Rightarrow OC^2 = CD^2 - OD^2 ....(ii)$

$LHS = OA^2 + OC^2$

$= AD^2 - OD^2 + CD^2 - OD^2 \qquad [from (i) and (ii)]$

$= AD^2 + CD^2 - 2OD^2$

$= AD^2 + AD^2 - 2\left(\frac{BD}{2}\right)^2 \qquad \left[AD = CD \text{ and } OD = \frac{BD}{2}\right]$

$= 2AD^2 - \frac{BD^2}{2}$

$= RHS$

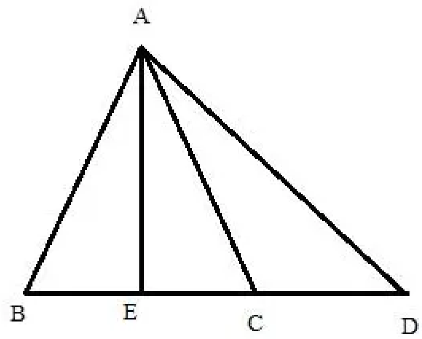
Solution 11:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
We consider the ΔACD and applying Pythagoras theorem we get,

$$\begin{aligned} AC^2 &= AD^2 + DC^2 \\ &= (AB^2 - DB^2) + (DB + BC)^2 \\ &= BC^2 - DB^2 + DB^2 + BC^2 + 2DB \cdot BC \quad (\text{Given}, AB = BC) \\ &= 2BC^2 + 2DB \cdot BC \\ &= 2BC(BC + DB) \\ &= 2BC \cdot DC \end{aligned}$$

Hence Proved.

Solution 12:



In an isosceles triangle ABC; AB = AC and D is point on BC produced. Construct AE perpendicular BC.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
We consider the rt. angled ΔAED and applying Pythagoras theorem we get,

$$AD^2 = AE^2 + ED^2$$
$$AD^2 = AE^2 + (EC + CD)^2 \quad ......(i)$$
$$[\because ED = EC + CD]$$

Similarly, in ΔAEC,

$$AC^2 = AE^2 + EC^2$$
$$AE^2 = AC^2 - EC^2 \quad ......(ii)$$
putting $AE^2 = AC^2 - EC^2$ in (i), we get,
$$AD^2 = AC^2 - EC^2 + (EC + CD)^2$$
$$= AC^2 + CD(CD + 2EC)$$
$$AD^2 = AC^2 + BD \cdot CD \quad [\because 2EC + CD = BD]$$
Hence Proved

Solution 13:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
We consider the rt. angled ΔACD and applying Pythagoras theorem we get,

$$CD^2 = AC^2 + AD^2$$
$$CD^2 = AC^2 + (AB + BD)^2 \quad [\because AD = AB + BD]$$
$$CD^2 = AC^2 + AB^2 + BD^2 + 2AB \cdot BD \quad ......(i)$$

Similarly, in ΔABC,

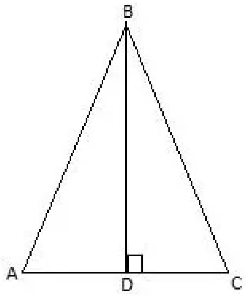
$$BC^2 = AC^2 + AB^2$$
$$BC^2 = 2AB^2 \quad [AB = AC]$$
$$AB^2 = \frac{1}{2}BC^2 \quad ......(ii)$$

Putting, AB^2 from (ii) in (i) we get,

$$CD^2 = AC^2 + \frac{1}{2}BC^2 \quad + BD^2 + 2AB \cdot BD$$
$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot (AD - AB)$$
$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot AD - 2AB^2$$
$$CD^2 - BD^2 = 2AB \cdot AD$$
$$DC^2 - BD^2 = 2AB \cdot AD$$

Hence Proved.

Solution 14:



In right angled ΔADB

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2 ....(i)$

$AC = AD + DC$

$\Rightarrow AC^2 = (AD + DC)^2$

$\Rightarrow AC^2 = AD^2 + DC^2 + 2AD \times DC$

$\Rightarrow AC^2 = AB^2 - BD^2 + DC^2 + 2AD \times DC \quad [from (i)]$

$\Rightarrow AC^2 = AC^2 - BD^2 + DC^2 + 2AD \times DC \quad [AB = AC]$

$\Rightarrow BD^2 - DC^2 = 2AD \times DC$

Solution 15:

Here,

$BD : DC = 1 : 3$

$\Rightarrow BD = \frac{1}{4} BC \text{ and } CD = \frac{3}{4} BC$

$AC^2 = AD^2 + CD^2 \text{ and } AB^2 = AD^2 + BD^2$

Therefore,

$$AC^2 - AB^2 = CD^2 - BD^2$$

$$= \left(\frac{3}{4} BC\right)^2 - \left(\frac{1}{4} BC\right)^2$$

$$= \frac{9}{16} BC^2 - \frac{1}{16} BC^2$$

$$= \frac{1}{2} BC^2$$

$\therefore 2AC^2 - 2AB^2 = BC^2$

$2AC^2 = 2AB^2 + BC^2$

Hence proved